

Beecher Community School District

The Vision for Ambitious Teaching of Mathematics Instruction

The [Michigan] Standards call for students to engage in rigorous content work, developing both conceptual and procedural understandings of mathematics and applying their understanding to solve authentic problems. In order for all students to develop this in-depth knowledge of mathematics, teachers must teach ambitiously.

Our vision of ambitious mathematics instruction is guided by a set of principles about children and the work of teaching. These include viewing children as sense-makers and knowing students as individuals and learners. Teachers must provide all students with equitable access to rigorous academic work and be responsive to the requirements of the school environment while at the same time working to improve schools as institutions in a democracy.

Core practices of ambitious teaching in mathematics, include:

- eliciting and responding to student reasoning,
- orienting students to each others' ideas and to the mathematical goal,
- setting and maintaining expectations for student participation,
- positioning students competently,
- teaching towards an instructional goal,
- assessing students' understanding, and
- using mathematical representations.

In order to create these mathematically rich classrooms, NCTM recommends the following teaching practices:

1. Establish mathematics goals to focus learning.

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

2. Implement tasks that promote reasoning and problem solving.

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

3. Use and connect mathematical representations.

Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

4. Facilitate meaningful mathematical discourse.

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

5. Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

6. Build procedural fluency from conceptual understanding.

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

7. Support productive struggle in learning mathematics.

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

8. Elicit and use evidence of student thinking.

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

What can Students do as a Result of Ambitious Math Instruction?

Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skills and guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (NCTM Vision for Mathematics Instruction)

1 — Make Sense of Problems and Persevere in Solving Them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

2 — Reason Abstractly and Quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize to —abstract a given situation and represent it symbolically and manipulate the

representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students confidently engage and persevere in complex mathematical tasks, make sense of quantities and their relationships in problem situations.

3 — Construct Viable Arguments and Critique the Reasoning of Others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 — Model With Mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 — Use Appropriate Tools Strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

6 — Attend to Precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 — Look For and Make Use of Structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.

8 — Look For and Express Regularity in Repeated Reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts while maintaining oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

REFERENCES

National Council on Teaching of Mathematics (NCTM)

Teacher Education by Design (TEDD) is a project of The University of Washington's College of Education supported by the Bill and Melinda Gates Foundation,
http://tedd.org?tedd_activity=ambitious-math-teaching

MI Excel Vision Statement for Ambitious, High Quality Subject Specific Instruction, 2016

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